Statistical Orbit Determination

Lecture 3 – Orbital Mechanics Review B

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Recap

- Lecture 2A – Notes posted here
  - Two body problem
  - Orbital Elements
    - Intro to some coordinate systems
- Problem solution has been posted
  - Quick review at beginning of this lecture
- Questions
  - Post them to lecture page
- Additional notes
  - Website revamp

Agenda

- Problem review
- Coordinate Systems
  - Definition
  - Transformation
- Perturbing accelerations
  - Conservative
  - Gravitational models
- Practice problem

Problem review

Time	0.0	1.0	2.0	3.0	4.0
Range, ρ	7.000000000	8.003905970	8.944271910	9.801147892	10.630145813
Calculated Range, $\hat{\rho}$	9.013878189	9.73203473	10.6004717	11.5815586	12.66688596
X_0	1.5	3.7	5.9	8.1	10.3
Y_0	10.0	10.35	10.4	10.15	9.6
$\dot{X}_0$	2.2	2.2	2.2	2.2	2.2
$\dot{Y}_0$	0.5	0.2	-0.1	-0.4	-0.7
g	0.3	0.3	0.3	0.3	0.3
X_s	1.0	1.0	1.0	1.0	1.0
Y_s	1.0	1.0	1.0	1.0	1.0

Problem review

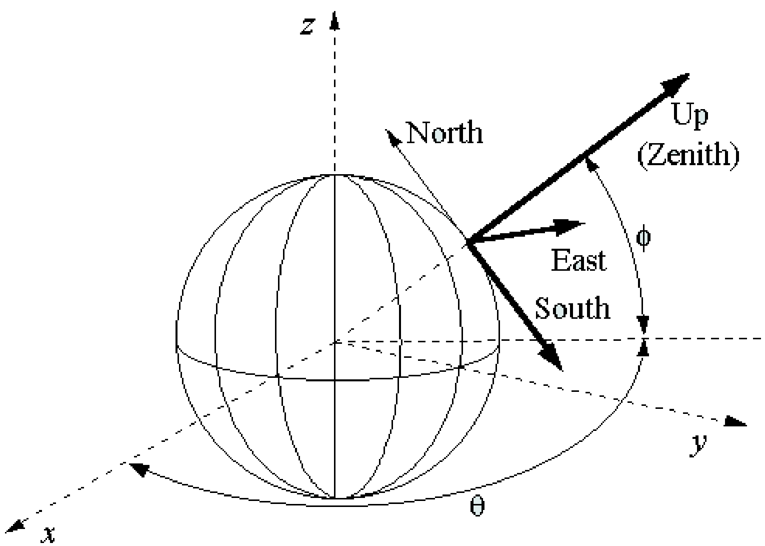
C:\Users\simps\Documents\GitHub\StatisticalOrbitDetermination\Soln-HW1-SimpsonAerospace\Debug\Soln-HW1-SimpsonAerospace.exe

```
Error for each iteration...
    2.34283
   0.567827
  0.0633679
0.000279296
6.94741e-09
6.94741e-09
Each iterations output...
    0.402014     0.940953      0.99995      1.00007      1.00007 -6.27744e+66
     8.04402      8.02017      8.00025            8            8 -6.27744e+66
      2.1321      2.01002            2      1.99998      1.99998 -6.27744e+66
     1.12213      1.00554     0.999945     0.999982     0.999982 -6.27744e+66
    0.556422      0.50253     0.499986     0.499993     0.499993 -6.27744e+66
```

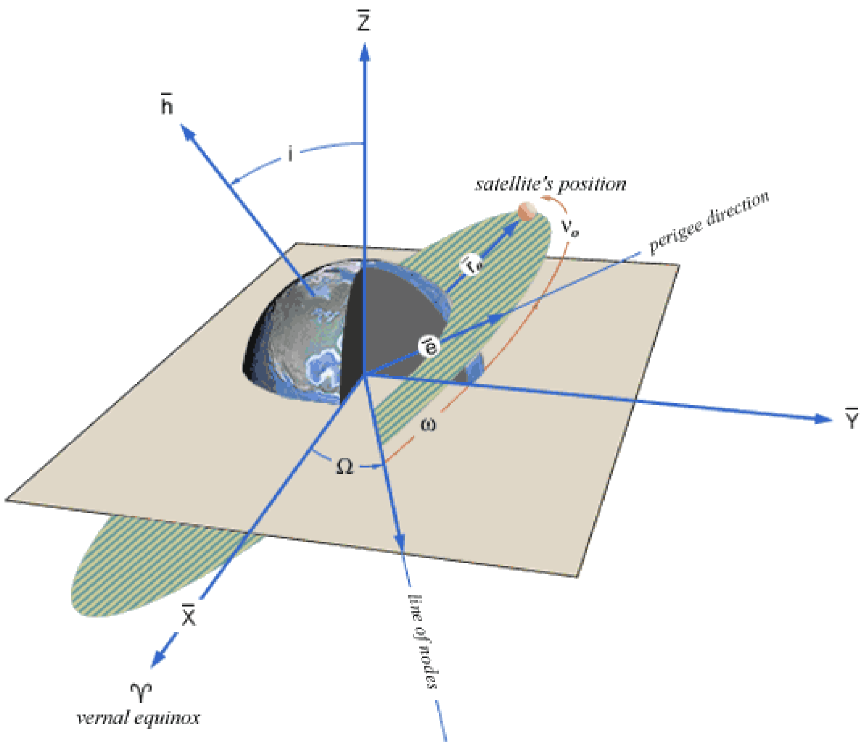
$$\delta u = (H^T H)^{-1} H^T \delta\rho$$

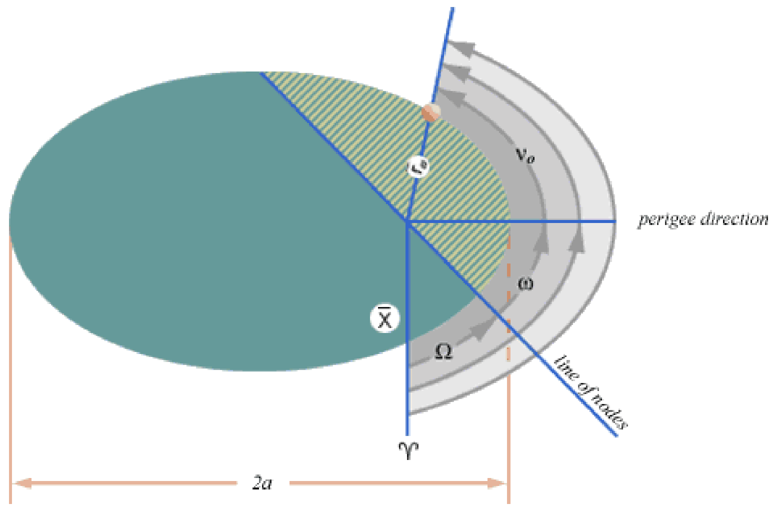
Coordinate Systems – Definition (1/4)

- Topocentric-Horizon Coordinate System
  - Fundamental plane is horizon
  - X points South, Y points East, and Z points up



Coordinate Systems – Definition (2/4)

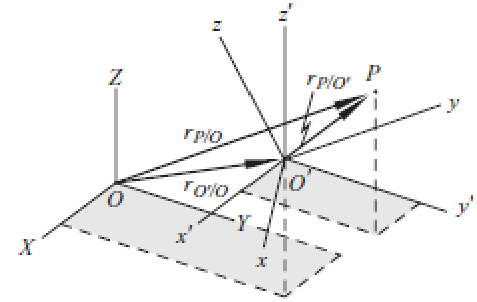




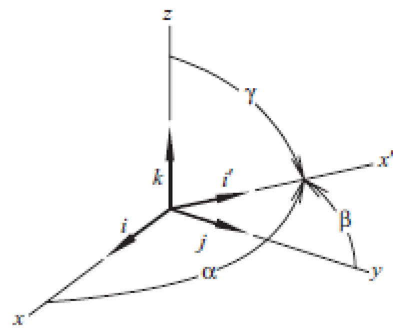
a - defines the size of the orbit
e - defines the shape of the orbit
i - defines the orientation of the orbit with respect to the Earth's equator.
ω - defines where the low point, perigee, of the orbit is with respect to the Earth's surface.
Ω - defines the location of the ascending and descending orbit locations with respect to the Earth's equatorial plane.
ν - defines where the satellite is within the orbit with respect to perigee.

Coordinate Systems – Definition (3/4)

- Three orthonormal vectors
  - Intersection is origin
  - Absolute (inertial) reference frame
- Coordinate transformations
  - Translation
    - $\boldsymbol{r}_{P/O} = \boldsymbol{r}_{O'/O} + \boldsymbol{r}_{P/O'}$
  - Rotation
    - $\hat{\imath}' = (\hat{\imath}' \cdot \hat{\imath})\hat{\imath} + (\hat{\imath}' \cdot \hat{\jmath})\hat{\jmath} + (\hat{\imath}' \cdot \hat{k})\hat{k}$
    - $\hat{\imath}' = \cos\alpha\,\hat{\imath} + \cos\beta\,\hat{\jmath} + \cos\gamma\,\hat{k}$



Translation



Rotation

Coordinate Systems – Definition (4/4)

- Attitude coordinates
  - Completely describe orientation of rigid body relative to reference
  - A set of coordinates $\{x_1, x_2, \ldots, x_n\}$
- Translational and orientation
  - Translational coordinate systems
    - Cartesian, polar, spherical, etc.
    - Differ in distance
    - Can grow infinitely
  - Attitude coordinate systems
    - DCM, Rodriguez parameters, Euler angles, etc.
    - Never further than 180° away

Coordinate Systems – Transformation

- Coordinate Transformations
  - $\hat{\imath}' = (\hat{\imath}' \cdot \hat{\imath})\hat{\imath} + (\hat{\imath}' \cdot \hat{\jmath})\hat{\jmath} + (\hat{\imath}' \cdot \hat{k})\hat{k}$
  - $\hat{\imath}' = \cos\alpha_1\,\hat{\imath} + \cos\beta_1\,\hat{\jmath} + \cos\gamma_1\,\hat{k}$
  - $\hat{\jmath}' = (\hat{\jmath}' \cdot \hat{\imath})\hat{\imath} + (\hat{\jmath}' \cdot \hat{\jmath})\hat{\jmath} + (\hat{\jmath}' \cdot \hat{k})\hat{k}$
  - $\hat{\jmath}' = \cos\alpha_2\,\hat{\imath} + \cos\beta_2\,\hat{\jmath} + \cos\gamma_2\,\hat{k}$

$$F_1 = \begin{bmatrix} C_{\alpha_1} & C_{\beta_1} & C_{\gamma_1} \\ C_{\alpha_2} & C_{\beta_2} & C_{\gamma_2} \\ C_{\alpha_3} & C_{\beta_3} & C_{\gamma_3} \end{bmatrix} F_2$$

- Minimum of 3 coordinates required
  - DCM – 9 independent parameters
  - Euler angle – 3
  - Quaternion – 4

Perturbing accelerations – Conservative (1/2)

- Acceleration of satellite with perturbing accelerations

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\,\vec{r} = \ddot{\vec{r}}_p$$

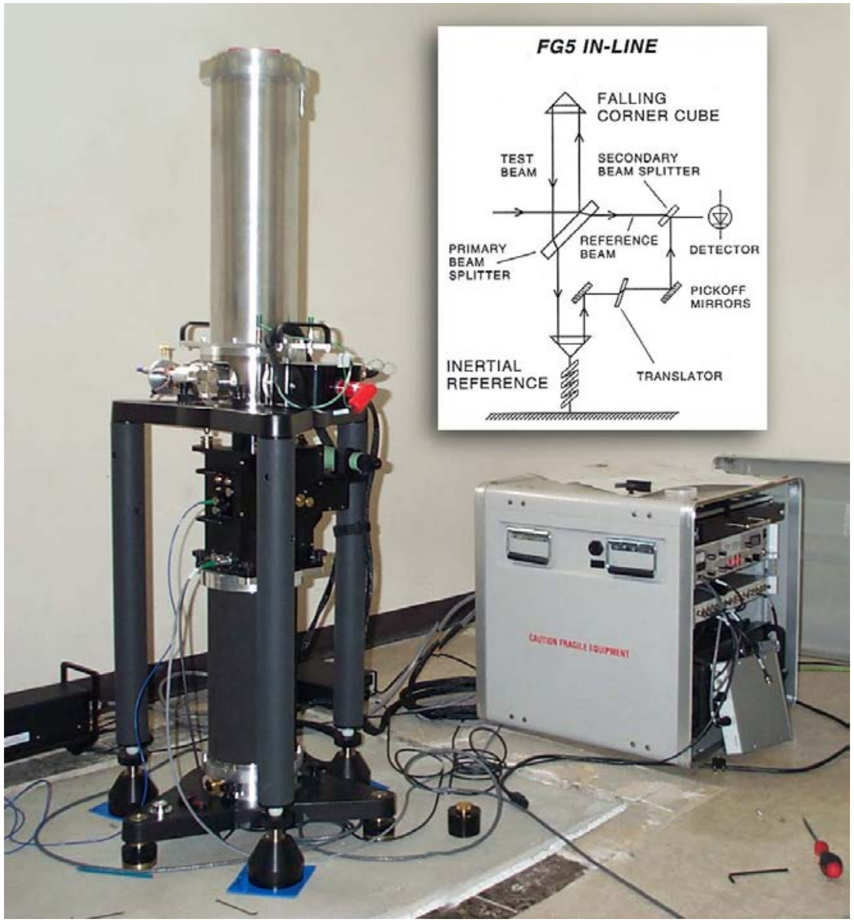
- Perturbations are conservative if only a function of position
  - Satellite does not lose nor gain mechanical energy
  - Exchanges energy between kinetic energy and potential energy
  - Specific mechanical energy is unique for each orbit
- Examples of non-conservative perturbations (changes to $\vec{r}, \vec{v}$)
  - Atmospheric drag
  - Outgassing
  - Tidal effects

Perturbing accelerations – Conservative (2/2)

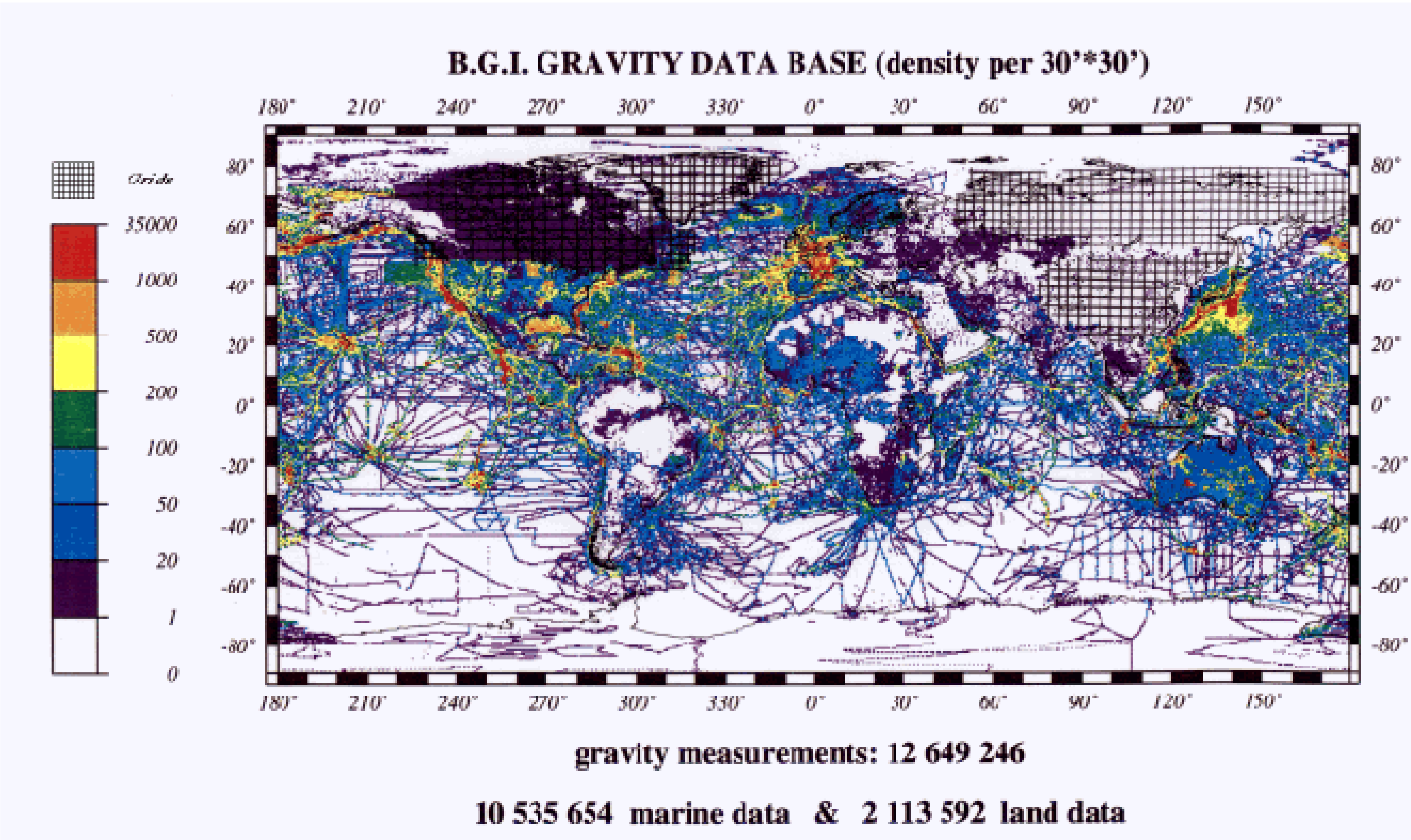
- Examples of conservative perturbations
  - N-body (celestial body) attractions
  - Nonspherical celestial bodies
  - Solar-radiation pressure
- Focus on the gravitational field effects
  - Nonspherical celestial bodies
  - Tidal effects
  - N-body attractions

Perturbing accelerations – Gravitational Models (1/6)

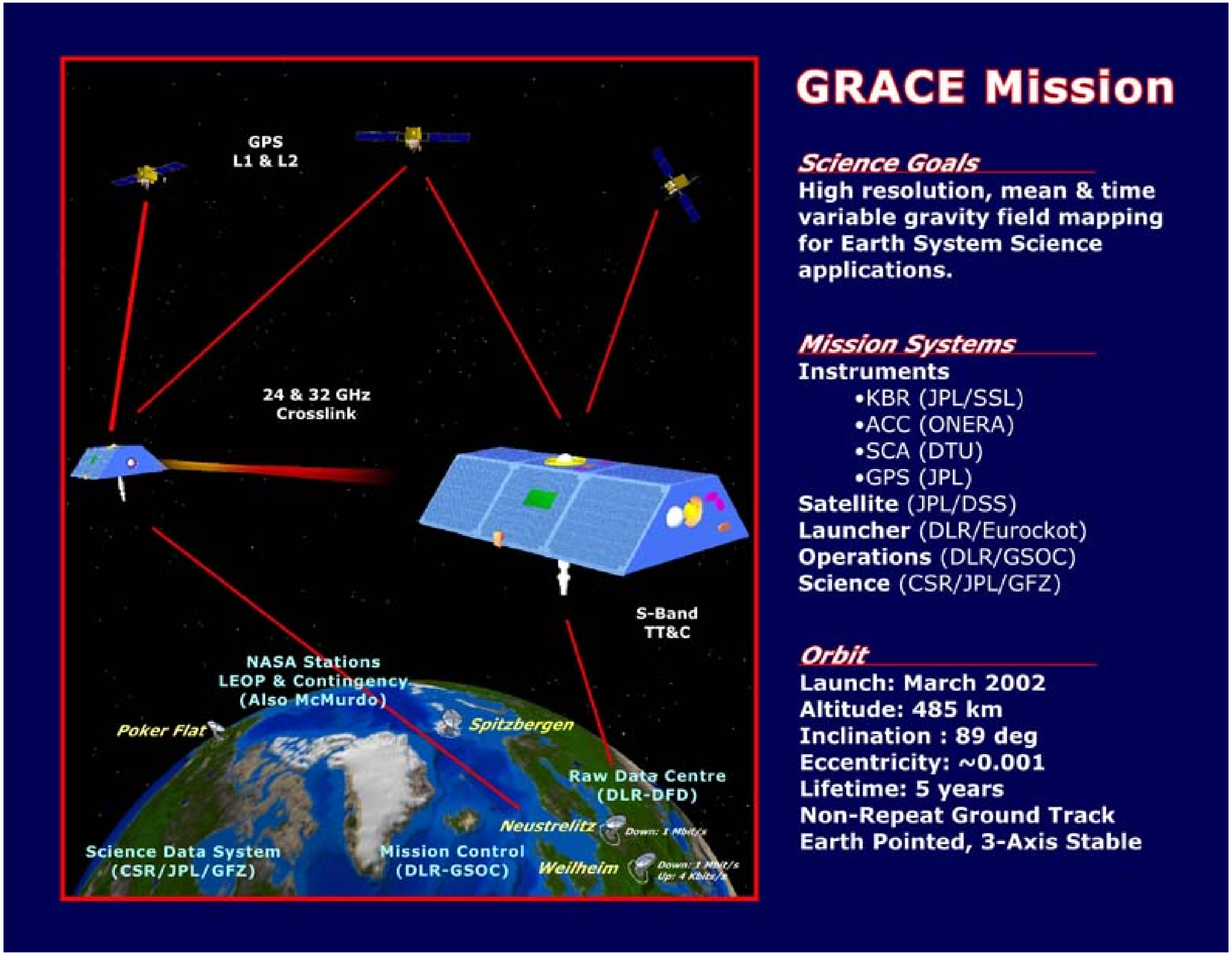
- Terrestrial Measurements



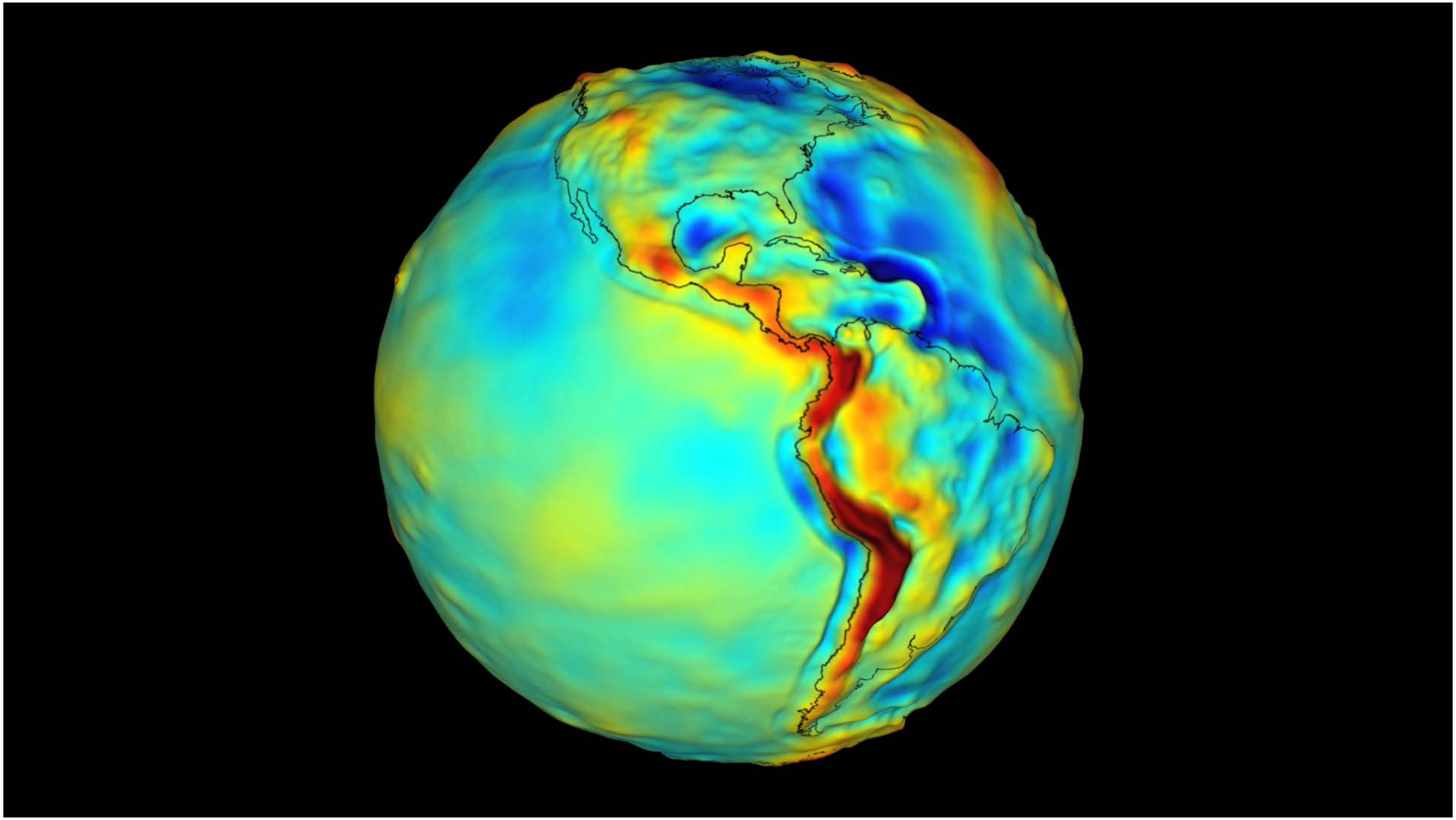
Perturbing accelerations – Gravitational Models (2/6)



Perturbing accelerations – Gravitational Models (3/6)

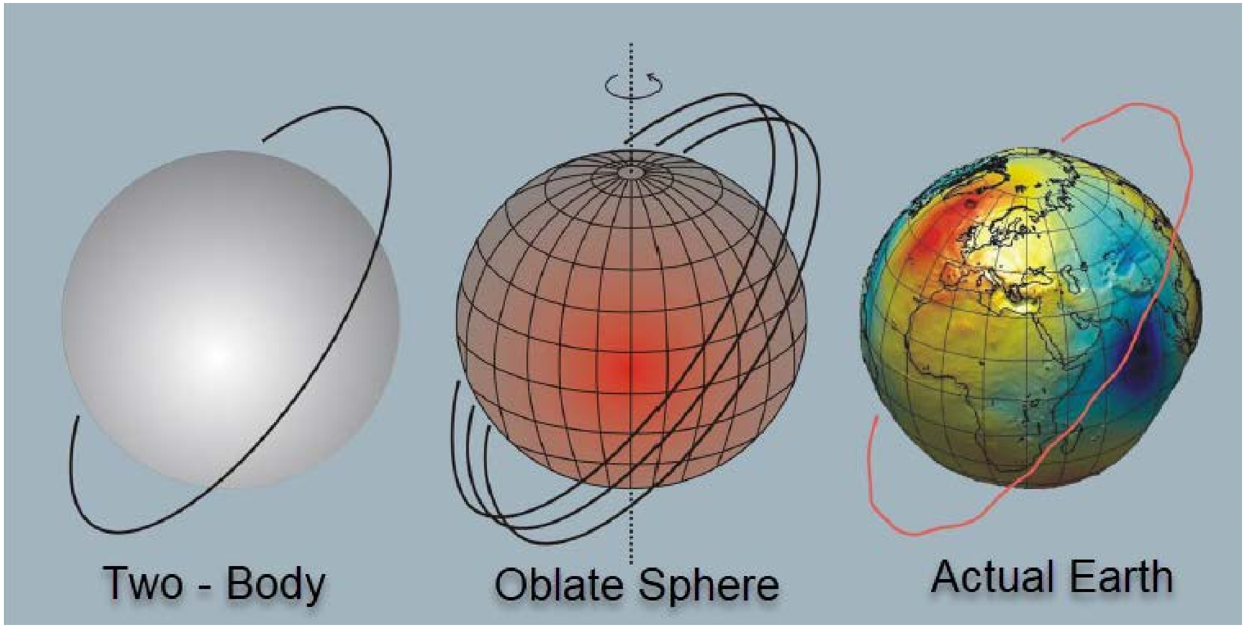


Perturbing accelerations – Gravitational Models (4/6)



Perturbing accelerations – Gravitational Models (5/6)

- Earth's Oblateness $(J_{2,0})$
  - Bulging at the equator
  - ~400 times larger than the next term
  - When included in satellite orbits maintains reasonable accuracy



Perturbing accelerations – Gravitational Models (6/6)

- Earth's bulge at equator pulls satellite down faster
  - Exerts a force component toward the equator
- Satellite reaches equator short of point for spherical Earth
  - East-bound satellite goes west
  - West-bound satellite goes east

$$\dot{\Omega} = -\frac{9.9358}{(1-e^2)^2}\left(\frac{r_{eq}}{r_{eq}+\bar{h}}\right)^{3.5} \cos i \ \ [\text{deg/mean solar day}]$$

- Secular motion of perigee too
  - Force is no longer proportional to inverse square radius

$$\dot{\omega} = \frac{9.9358}{(1-e^2)^2}\left(\frac{r_{eq}}{r_{eq}+\bar{h}}\right)^{3.5} \left(2 - \frac{5}{2}\sin^2 i\right) \ [\text{deg/mean solar day}]$$

Practice problem: Gibbsian method

LOST IN SPACE

Gibbsian Method – Introduction

- Obtain $\boldsymbol{r}, \boldsymbol{v}$ from three coplanar position vectors through successive measurements of $\boldsymbol{\rho}$, El, and Az.
  - Developed using pure vector analysis
  - Historically, first contribution of an American scholar to celestial mechanics
- Gibbs problem: Given three nonzero coplanar vectors $\boldsymbol{r_1}, \boldsymbol{r_2}$, and $\boldsymbol{r_3}$ which represent three sequential positions of an orbiting object on one pass, find the parameter p and the eccentricity e of the orbit and the perifocal base vectors $\boldsymbol{P}, \boldsymbol{Q}$, and $\boldsymbol{W}$

Gibbsian Method – Problem Statement

- Given three position vectors, $\boldsymbol{r_1}, \boldsymbol{r_2}, \text{and } \boldsymbol{r_3}$, find PQW (perifocal basis vectors expressed in the IJK system), the semi-latus rectum, eccentricity, period, and the velocity at position two.
  - $\boldsymbol{r_1} = 1.000\ \hat{k}$
  - $\boldsymbol{r_2} = -0.700\ \hat{j}\ - 0.8000\ \hat{k}$
  - $\boldsymbol{r_3} = 0.9000\ \hat{j} + 0.5000\ \hat{k}$