Statistical Orbit Determination

Lecture 6 – Coordinate Systems and Time

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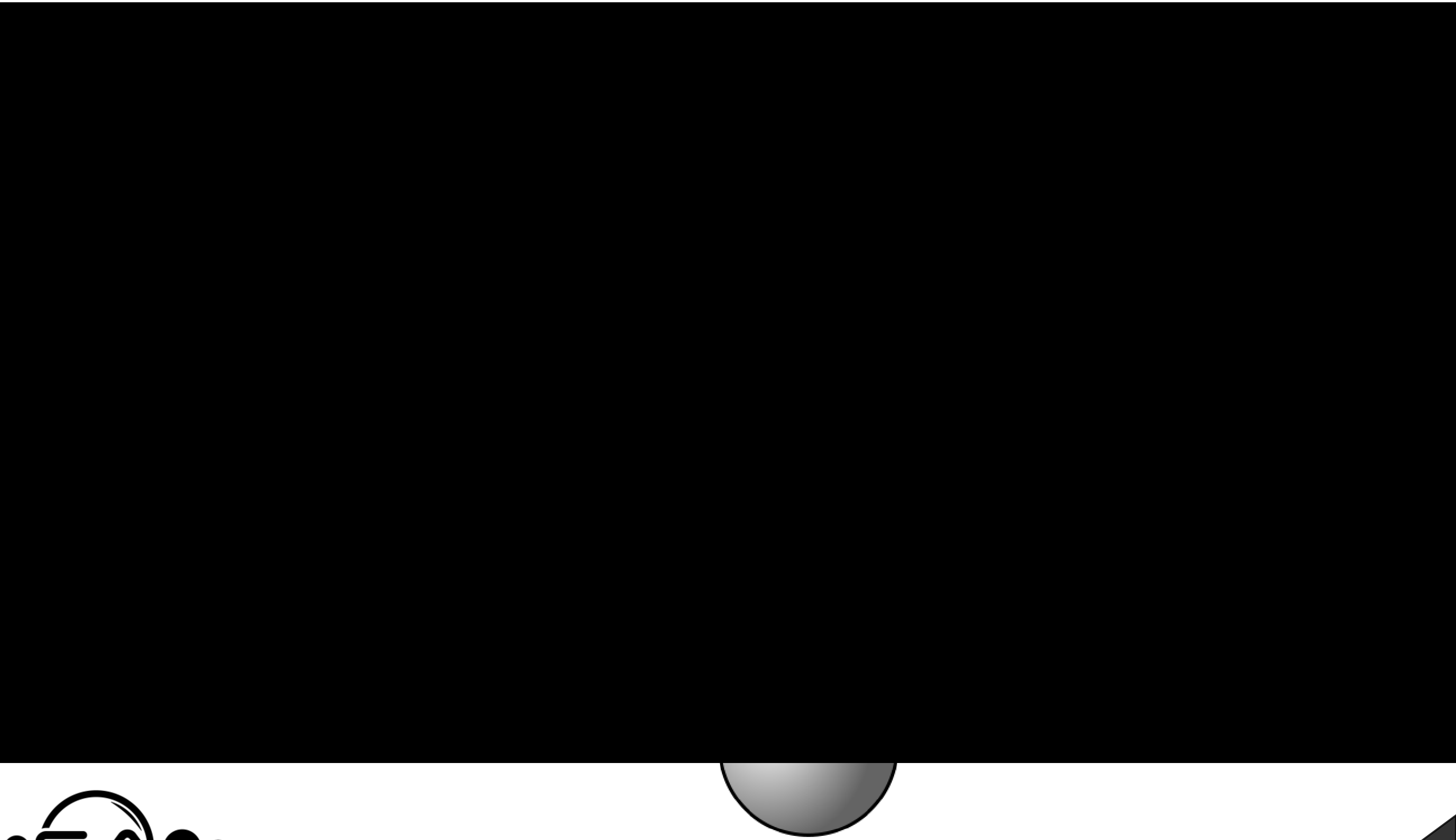
Recap

- Lecture 5 – Notes posted here
  - Perturbed Motion
- Questions
  - Post them to YouTube page

Agenda

- Coordinate Systems and Time: Introduction
  - Precession and Nutation
  - Earth Rotation and Time
  - Earth-Fixed and Topocentric Systems
  - Transformation between ECF and ECI
- Orbit Accuracy
- Assigned Problems

Reference Frame – Precession and Nutation



Reference Frame – Rotation and Time (1/5)

- Rotation, $\omega_\oplus$
  - Simplified model is fixed in direction and magnitude
  - Conical CCW motion, full revolution in 430 days, Chandler period
  - The polar motion is confined to 0.6 arcsec over decadal time scale
- Changes in magnitude and length of day
  - Change in $\omega_\oplus$ means change period required for rev of Earth w/r stars
  - Not w/r to the sun
  - Period change expressed in form of Universal Time, Δ(UT1)
  - Difficult to predict period change
  - Regular observation and reported by International Earth Rotation Service

Reference Frame – Rotation and Time (2/5)

- Estimation of Δ(UT1)
  - Separation of Δ(UT1) from orbit node, Ω, complicates estimation
  - Polar motion and satellite state can be used (GPS and Lageos)
    - Errors in Ω absorbed in estimation of Δ(UT1)
    - Cannot provide reliable long-term estimates of Δ(UT1)
- Time is different
  - Event uniquely identified by local time or Universal Time
  - Epoch J2000.0 is January 1, 2000, 12 hours
- Julian Date (JD)
  - Measured from 4713 b.c. and day begins at noon
  - J2000.0 is JD 2451545.0 days
    - January 1, 2000, 0 hours (midnight) would be JD 2451544.5

Reference Frame – Rotation and Time (3/5)

- Modified Julian Date (MJD)
  - JD minus 2400000.5
  - J2000.0 is MJD 51544.5 day
- Reference time necessary because independent variable
  - Every observation is time tagged using a reference clock
  - Reference clocks use oscillators
    - Typically crystal oscillators used
    - Affected by temperature and some temporal characteristics
    - High accuracy uses atomic frequencies

Reference Frame – Rotation and Time (4/5)

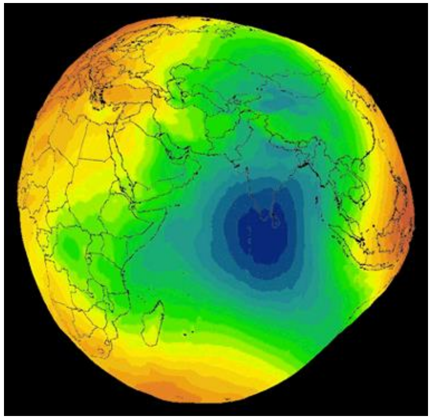
- Various time systems
  - Terrestial Dynamical Time (TDT)
    - Used for Earth satellites
    - Also known as terrestrial time (TT)
  - Barycentric Dynamical Time (TDB)
    - Used for solar system applications
  - International Atomic Time (TAI)
    - Both TDT and TDB related to TAI at a specified epoch
    - Based on cesium atomic clocks
  - Universal Time
    - Measure of time that is basis for all civil time-keeping
    - Coordinated Universal Time (UTC) derived from TAI where $UT1 = UTC + \Delta(UT1)$
    - UTC maintained by NIST and USNO
    - UTC(USNO) and TAI based on ensemble of cesium oscillators and hydrogen masers

Reference Frame – Rotation and Time (5/5)

- Difference between time systems
  - UTC needs leap seconds to maintain synchronization
    - UTC and Δ(UT1) require leap seconds for synchronization within ± 0.9 seconds
  - Constant difference between TAI and TT
    - TT – TAI $= 32.184$ seconds
    - TDB and TT difference is periodic function, relativistic effect
  - GPS time (GPS-T) related to TAI
    - Leap second adjustments applied to UTC are not used in GPS-T
    - Current leap second separation is 37.0 seconds as of Jan 1, 2017

Reference Frame – EF and Topocentric (1/3)

- Earth-fixed and topocentric systems
  - Earth-fixed frame not defined since not a rigid body
  - Mass deformation of luni-solar gravity changes coordinates on surface
  - True Earth-fixed frame does not exist
  - Terrestrial Reference Frame (TRF)
    - Origin is coincident with center of mass
    - "Attached," x-axis approx. coincident with Greenwich meridian
    - z-axis approx. coincident with $\omega_{\oplus}$
    - International Earth Rotation Service TRF or ITRF
    - WGS-84, used with many GPS applications



Reference Frame – EF and Topocentric (2/3)

- Ellipsoid of revolution not sphere
  - Spherical coordinates used to describe gravitational potential

$$x = r\cos\phi\cos\lambda$$
$$y = r\cos\phi\sin\lambda$$
$$z = r\sin\phi$$

  - ϕ, geocentric latitude, λ, longitude, and r is magnitude of pos vector
  - Alternate set used with ellipsoid
    - Geodetic latitude, ϕ', longitude, λ, and height above ellipsoid, h

$$x = (N_h + h)\cos\phi'\cos\lambda$$
$$y = (N_h + h)\cos\phi'\sin\lambda$$
$$z = (N_h + h - \bar{e}^2 N_h)\sin\phi'$$

    - Where eccentricity of the elliptical cross-section is

$$\bar{e}^2 = \bar{f}\left(2 - \bar{f}\right)$$
$$N_h = \frac{R_e}{(1 - \bar{e}^2\sin^2\phi')^{1/2}}$$

$$\bar{f} = \frac{R_e - R_p}{R_e}$$

$$x^2 + y^2 + \left(\frac{R_e}{R_p}\right)^2 z^2 = R_e^2$$

Reference Frame – EF and Topocentric (3/3)

- Topocentric
  - Northward, Eastward, local vertical point on a surface
    - x_t eastward, y_t northward, z_t local vertical
  - Earth-fixed in terms of topocentric system is

$$\bar{r}_t = T_t(\bar{r} - \bar{r}_s) = T_t\bar{\rho}$$

$$T_t = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}$$

  - Elevation and Azimuth

$$\sin(El) = \frac{z_t}{r_t} \quad -90° \le El \le 90°$$

$$\sin(Az) = \frac{x_t}{r_{xy}} \quad 0 \le Az \le 360°$$

$$\cos(Az) = \frac{y_t}{r_{xy}}$$

Reference Frame – Transform ECF and ECI

- Transform between ECF and ECI systems
  - Transformation matrix between ECF to J2000 system is complex
    - Must consider precession, nutation, polar motion, and UT1

$$T_{XYZ}^{xyz} = WS'NP$$

    - Where the transformation is from J2000 to ECF,
    - P applies to precession from epoch to current time
    - N applies nutation at current time
    - S' applies rotation to account for true sidereal time
    - W applies polar motion to align the z axis (true pole) with the pole of the ECF system

Orbit Accuracy

- Orbit accuracy
  - Accuracy of solution of EOM with parameters in model of forces
  - General perturbation technique concerned with small parameters
  - Special perturbation technique concerned with step size
  - Solution technique accuracy
    - Error introduced in the solution of the equations of motion by the solution technique
    - No consideration given to the accuracy of the parameters in the equations
  - Force model accuracy
    - Parameters in force models and modeling of forces most significant error source
    - All parameters in force model have been determined by some means
  - Specification of requirement
    - Equations of motion distinctly different if orbit must be determined with cm vs km accuracy

Practice problems: The Orbit Problem

PREDICT THE ORBIT

Assigned Problems - Overview

- You are given three problems involving orbital motion. They have been picked to ensure you have a sufficient understanding of orbital mechanics before proceeding. The problems resemble numbers 4, 5, 6, 10, 11, and 12 from the textbook.
- These problems should be complete by Friday, February 8.

Assigned Problems – Problem 1

- Given the following position and velocity of a satellite
  - Expressed in a non-rotating geocentric coordinate system

	Position (m)	Velocity (m/s)
X	7088580.789	-10.20544809
Y	-64.326	-522.85385193
Z	920.514	7482.075141

a) Determine the six orbital elements (a, e, i, Ω, ω, M_0)

b) Assuming X_0 is given and two-body motion, predict position and velocity at $t = 3{,}000$ sec. Determine flight path angle at this time.

c) Determine the latitude and longitude of the subsatellite point for $t = 3{,}000$ sec if α_G at $t = 0$ is 0. Assume the Z axis of the nonrotating system is coincident with the z axis of the rotating system.

Assigned Problems – Problem 2 (1/2)

- Orbit of CRISTA-SPAS-2
  - Cryogenic Infrared Spectrometers and Telescopes for the Atmosphere

The joint venture of DLR and NASA, the small free-flying satellite contains three telescopes, four spectrometers, and a GPS receiver on-board. It is deployed from the shuttle Discovery on STS-85 in August 1997. Using on-board navigation, the receiver measurements are processed in an Earth-centered, Earth-fixed coordinate system.

August 18, 1997

GPS-T (hrs:min:sec)	00:00:0.000000	00:00:03.000000
x	3325396.441	3309747.175
y	5472597.483	5485240.159
z	-2057129.050	-2048664.333

August 19, 1997

GPS-T (hrs:min:sec)	00:00:0.000000	00:00:03.000000
x	4389882.255	4402505.030
y	-4444406.953	-4428002.728
z	-2508462.520	-2515303.456

Assigned Problems – Problem 2 (2/2)

a) Demonstrate that the node location is not fixed in space and determine an approximate rate of node change (degrees/day) from these positions. Compare the node rate with the value predicted by

$$\dot{\Omega} = -\frac{3}{2} J_2 \, \frac{n}{(1-e^2)^2} \left(\frac{a_e}{a}\right)^2 \cos i$$

b) Determine the inclination of CRISTA-SPAS-2 during the first 3-sec interval and the last 3-sec interval.

Comment: The position vectors determined by GPS in this case are influenced at the 100-meter level by Selective Ability, but the error does not significantly affect this problem.

Assigned Problems – Problem 3 (1/2)

- GLONASS
  - Russia's answer for American GPS

Given a set of initial conditions for a high-altitude GLONASS satellite, numerically integrate the equations of motion for one day.

a) Assuming the satellite is influenced by J_2 only, derive the equations of motion in non-rotation coordinates. Assume the nonrotating Z axis coincides with the Earth-fixed z axis.

b) During the integration, compute the Jacobi constant and the Z component of the angular momentum. Are these quantities constant?

c) Plot the six orbital elements as a function of time.

d) Identify features similar to and different from Fig. 2.3.5

Assigned Problems – Problem 3 (2/2)

e) Compare the node rate predicted by

$$\dot{\Omega} = -\frac{3}{2} J_2 \frac{n}{(1-e^2)^2} \left(\frac{a_e}{a}\right)^2 \cos i$$

with a value estimated from (c).

f) Compare the amplitude of the semimajor axis periodic term with

$$a(t) = \bar{a} + 3\bar{n}\bar{a}J_2 \left(\frac{a_e}{\bar{a}}\right)^2 \sin^2 \frac{\bar{\imath}(\cos(2\omega + 2M))}{2\dot{\omega}_s + 2\dot{M}_s}$$

g) Plot the ground track. Does the ground track repeat after one day?

a	e	i	Ω	ω	M_0
25500.0 km	0.0015	63 deg	-60 deg	0 deg	0 deg