Statistical Orbit Determination

Lecture 10 – Conceptual Example

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Recap

- Lecture 9 – Notes posted here
  - Conceptual Measurements
- Questions
  - Post them to YouTube page

Agenda

- Example

Examples for range and range-rate

MEASUREMENT MODELING

Measurement Modeling - Overview

- Two-way ranging (p. 106)

  Consider satellite in equatorial posigrade circular orbit with an altitude of 600 km above a spherical Earth. Assume satellite is 20° in true anomaly past the zenith direction of a two-way ranging station, which places the satellite at 4.3° elevation w/r to the station. Assume a signal is transmitted from the station at $t = 0$.

h	e	i	f	El
600 km	0	0 deg	20 deg	4.3 deg

- Range rate

  Consider the same satellite with a transmitter beacon. Assume the transmitter is operating in the Ka-band at 24.25 GHz.

- Solution on GitHub

Measurement Modeling – Two-way (1/3)

h	e	i	f	El
600 km	0	0 deg	20 deg	4.3 deg

- Two-way ranging (p. 106)
  - Geometry
    - Determine ideal range between ground station and satellite
    - $\hat{z}_t \cdot \bar{r} = \cos 4.3°$ and $\left(\hat{z}_t \cos i_{gs}\right) \cdot \bar{r} = \cos 20°$

$$\cos i_{gs} \left(\hat{z}_t \cdot \bar{r}\right) = \cos 20°$$
$$i_{gs} = \phi = \pm 19.55°$$

    - Assuming longitude, λ, is $0°$
    - $\rho(t = 0) = 2.344 \times 10^6$ meters

Measurement Modeling – Two-way (2/3)

h	e	i	f	El
600 km	0	0 deg	20 deg	4.3 deg

- Two-way ranging (p. 106)

$$\rho_{rt} = c(T_R - T_T) + b(T_R - T_T) + \delta\rho_{atm} + \epsilon$$

  – Computed range requires iterative process
    - – Satellite signal arrival time is unknown
    1. Find instantaneous ρ at time t_T (assume negligible clock errors)
    2. Signal arrival, $t_a = t_T + \rho/c$
    3. New range, ρ_{new} at t_a, and position of station
    4. Compare ρ_{new} and ρ difference
    - – Can estimate by halving instaneous range and accounting for speed of light
    - – Same with altimeter

$$h_{avg} = \frac{h_{rt}}{2}$$

$$t_{avg} = t_T + h_{avg}/c$$

Measurement Modeling – Two-way (3/3)

- Two-way ranging (p. 106)
  - Expected answers

$$\rho_{t=0} = 2343532.4 \text{ meters}$$
$$\rho_{t=0.007817} = 2343864.4 \text{ meters}$$
$$\rho_{rt} \approx 4687396.8 \text{ meters}$$

Measurement Modeling – Range rate (1/2)

h	e	i	ν	El	$f_T = f_G$
600 km	0	0 deg	20 deg	4.3 deg	24.25 GHz

- Satellite transmits signal with known frequency, f_T
- Received signal mixed with reference f_G
  - Receiver designed to count number of cycles between t_{R1} and t_{R2}

$$N_{1,2} = \int_{t_{R1}}^{t_{R2}} (f_G - f_R)\, dt$$

$$N_{1,2} = (f_G - f_T)(t_{T2} - t_{T1}) + \frac{f_G(\rho_2 - \rho_1)}{c}$$

$$\frac{N_{1,2}}{\delta t} = \left(\frac{f_T}{c}\right)\left(\frac{\delta\rho}{\delta t}\right)$$

- Received frequency depends on range rate

$$f_R = f_T - \frac{N_{1,2}}{\delta t} = f_T\left(1 - \left(\frac{\delta\rho}{\delta t}\right)/c\right)$$

Measurement Modeling – Range rate (2/2)

h	e	i	ν	El	$f_T = f_G$
600 km	0	0 deg	20 deg	4.3 deg	24.25 GHz

- Deciding that $\delta t = 1$ ms
  - Doppler count: 133965665.930066
  - Received frequency: -109.715666 GHz
  - Apparent frequency is greater than actual frequency.
  - The satellite is moving towards the ground station.
  - (f_T - f_R)>f_T, (f_T-f_R) = 133.965666 GHz
  - Range estimate at t_{R2}: 9983046.823160