Statistical Orbit Determination

Lecture 11 – Real Measurements

Presenter: Christopher R. Simpson

Recap

- Lecture 10 – Notes posted here
  - Example Conceptual Measurements
- Questions
  - Post them to YouTube page

Agenda

- Environment
  - Atmosphere
  - Relativity
- Measurement systems
  - One way ranging
  - Two way ranging
  - Doppler measurements
  - Differenced measurements

Environment – Atmosphere (1/2)

- Troposphere
  - Refractive to radio
    - Troposphere delay, even though stratosphere 20%
    - $\delta\rho = 10^{-6} \int N\, ds$, where N is refractivity
    - Dry refractive component contributes 90%
    - $N_d = 77.6(P/T)$ where pressure is in mb
    - $N_w = 3.73 \times 10^5 (e/T^2)$ where e is par. water vapor pressure
  - Dispersive to optical
  - Dependent on elevation

Environment – Atmosphere (2/2)

- Troposphere
  - Dispersive to radio
    - $\delta t = \alpha / f^2$, where α represents TEC
    - α is positive for group delays (psuedorange) and negative for carrier phase
    - Transmitting on two different f removes ionosphere for OD
    - $\rho_c = \gamma_1 \rho_1 - \gamma_2 \rho_2 \mid \ \gamma_1 = \frac{f_1^2}{f_1^2 - f_2^2} \mid \gamma_2 = \frac{f_2^2}{f_1^2 - f_2^2}$
  - No significant effect to optical
  - Dependent on TEC
    - Temporal and spatially dependent
    - Starts at 80km and upward

Environment – Relativity

- General Relativity
  - Satellite experience relativistic precession of perigee
    - $e = 0$, clock beats faster or slower than gnd, dependent on altitude
  - Clock varies w/ same orbital period for $e \neq 0$
    - Still varies about mean frequency
    - Time offset correction, $\Delta t_r = 2\bar{r} \cdot \bar{v}/c^2$
    - Time delay caused by Earth, $\Delta t_{delay} = \frac{2\mu}{c^3} \ln\left[\frac{r_1 + r_2 + \rho}{r_1 + r_2 - \rho}\right]$
    - Sagnac effect only concern for high precision clock transfer via sat

Measurement Systems – One Way Range (1/2)

- Global Positioning System (GPS)
  - GPS constellation, 6 orbital planes $(i = 55°)$, $T = 11$hr 58min
  - 3 Civil frequencies used
    - L1 and L2 transmit at 1575.42 and 1227.60 MHz, respectively
    - Transmit on carrier frequencies, $154f_0$ and $120f_0$, where $f_0 = 10.23$ MHz
    - Third frequency, L5 is $115f_0$
  - Information superimposed on carrier
    - PRN is transmitted as bits on over carrier
    - Documented determination of bits attaches time to each
    - Cross-correlation with generated signal on ground with received signal
    - Codes transmitted are (C/A, 1023 bits, 1ms), (P, 37 weeks, individual), (Y, AS)

Measurement Systems – One Way Range (2/2)

- Psuedorange measurement from L1 or L2
  - $\tilde{\rho} = \rho + c(\delta t_R - \delta t_T) + \delta\rho_{trop} + \delta\rho_{ion} + \epsilon$
  - True range requires true GPS position and receiver coordinates and true times
  - IN practice, may not be known with high accuracy
- Carrier phase gives more precise measurement
  - $\phi = \rho + c(\delta t_R - \delta t_T) + \lambda\widetilde{N} + \delta\rho_{trop} - \delta\rho_{ion} + \epsilon$
  - ϕ, is measured phase range
  - $\widetilde{N}$, is integer phase ambiguity