Statistical Orbit Determination

Lecture 9 – Conceptual Measurement Systems

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Recap

- Lecture 8 – Notes posted here
  - Simulating Ideal Measurements
- Questions
  - Post them to YouTube page

Agenda

Ideal Observations – Range

- Ideal Range
  - Ideal means ignore propagation
  - Instantaneous range or geometric range
    - Propagation and other errors captured in the observed range
  - Difference between instrument and satellite position vector

$$\rho = [(\bar{r} - \bar{r}_I) \cdot (\bar{r} - \bar{r}_I)]^{1/2}$$

  - Observed range,

$$\rho_{obs} = \rho + \epsilon$$

- Geometric range is invariant between different frames
  - ρ will be identical between both ECF and J2000
    - Magnitude of difference in position vectors

$$\rho = [(X - X_I)^2 + (Y - Y_I)^2 + (Z - Z_I)^2]^{1/2}$$

$$\rho = [(x - x_I)^2 + (y - y_I)^2 + (z - z_I)^2]^{1/2}$$

Ideal Observations – Range rate

- Ideal range rate
  - Differentiating the range with respect to time

$$\dot{\rho} = \frac{\bar{\rho} \cdot \dot{\bar{\rho}}}{\rho}$$

$$\rho = \left[(X - X_I)(\dot{X} - \dot{X}_I) + (Y - Y_I)(\dot{Y} - \dot{Y}_I) + (Z - Z_I)(\dot{Z} - \dot{Z}_I)\right] \;\; /\rho$$

  - Relative velocity in direction defined by ρ
    - Range-rate is the component of the relative velocity between the observing instrument and the satellite in the line-of-sight direction

$$\dot{\rho}_{obs} = \dot{\rho} + \epsilon$$

- Azimuth and elevation

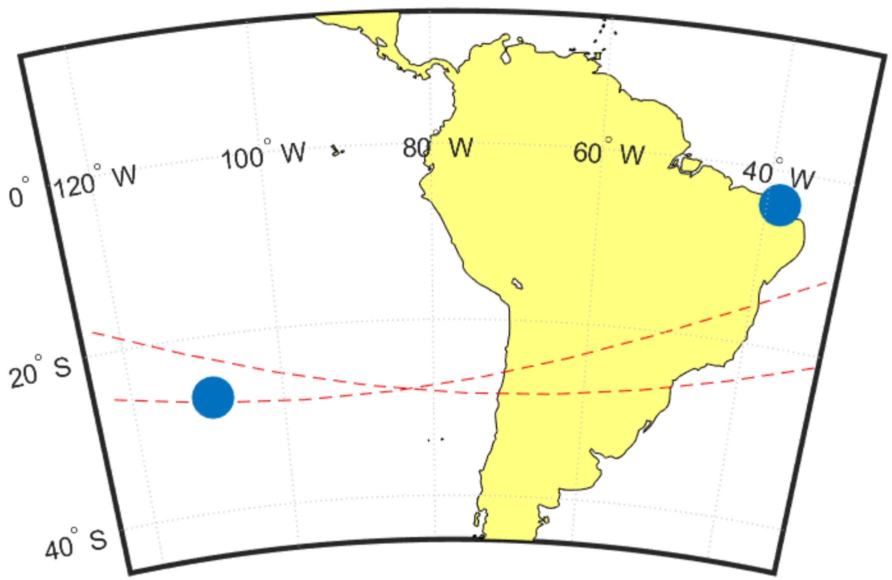
$$\sin(El) = \frac{z_t}{r_t} \quad -90° \leq El \leq 90°$$

$$\sin(Az) = \frac{x_t}{r_{xy}} \quad 0 \leq Az \leq 360°$$

$$\cos(Az) = \frac{y_t}{r_{xy}}$$

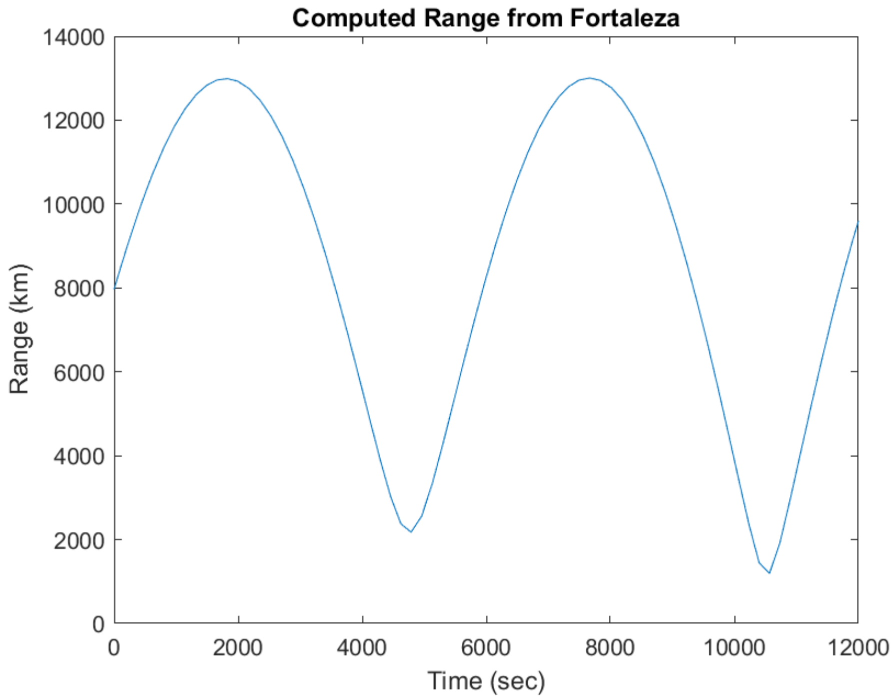
Ideal Observations – Simulated Obs (1/4)

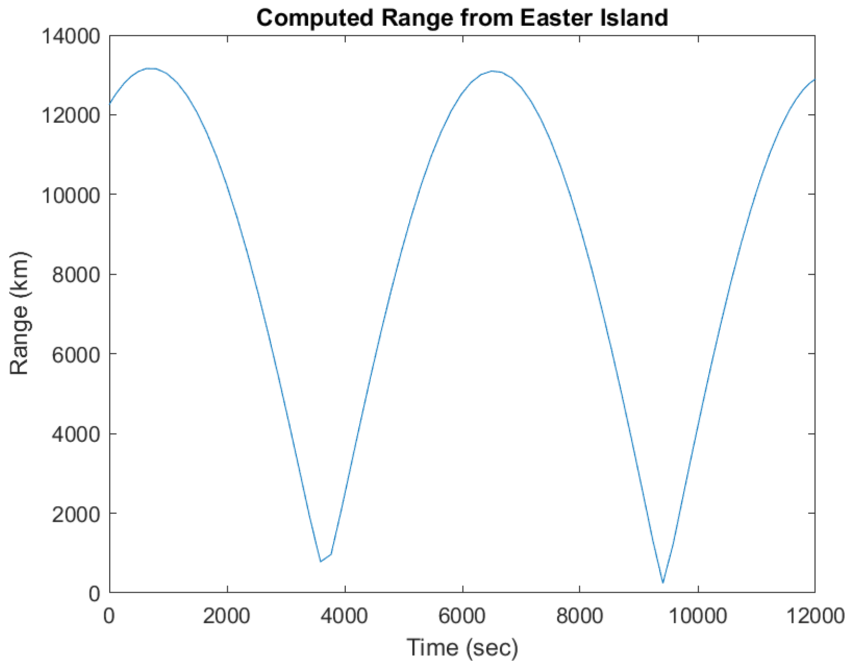
- Simulation of ideal observations
  - Set of initial conditions represent the “truth” then simulate observations
    - Use geometric range and range rate in this example
  - Two sites, Easter Island and Fortaleza, Brazil



Ideal Observations – Simulated Obs (2/4)

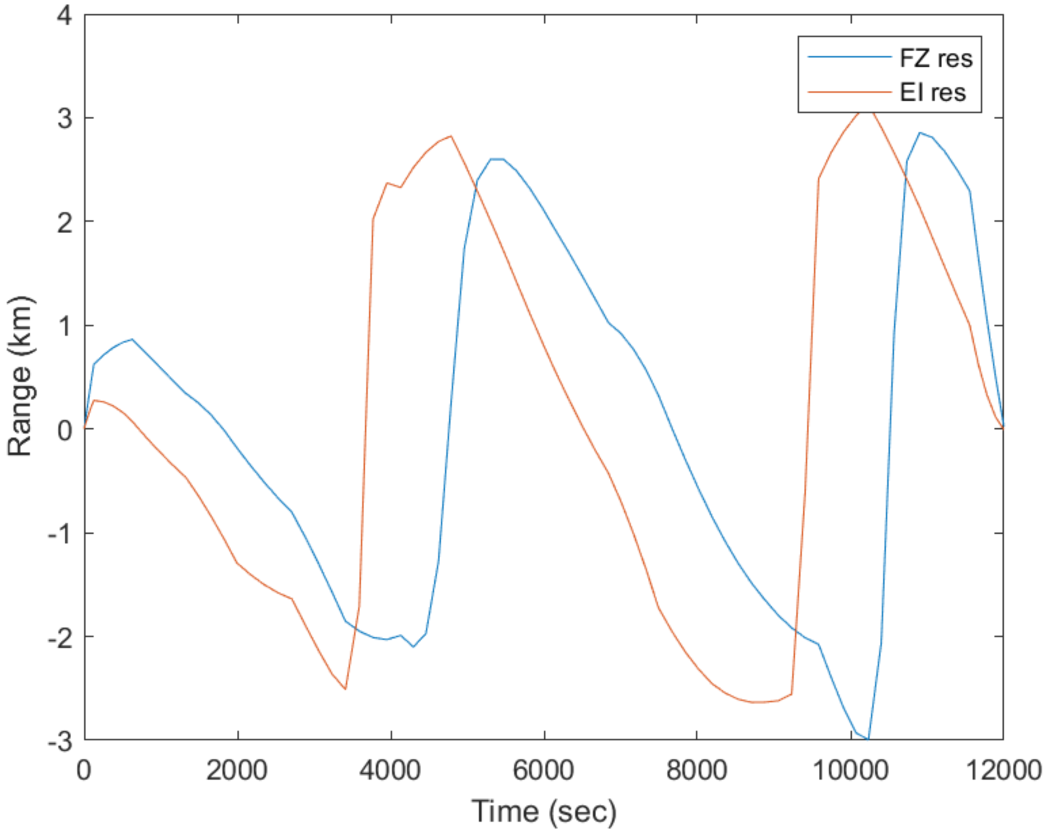
- Range from Easter Island and Fortaleza





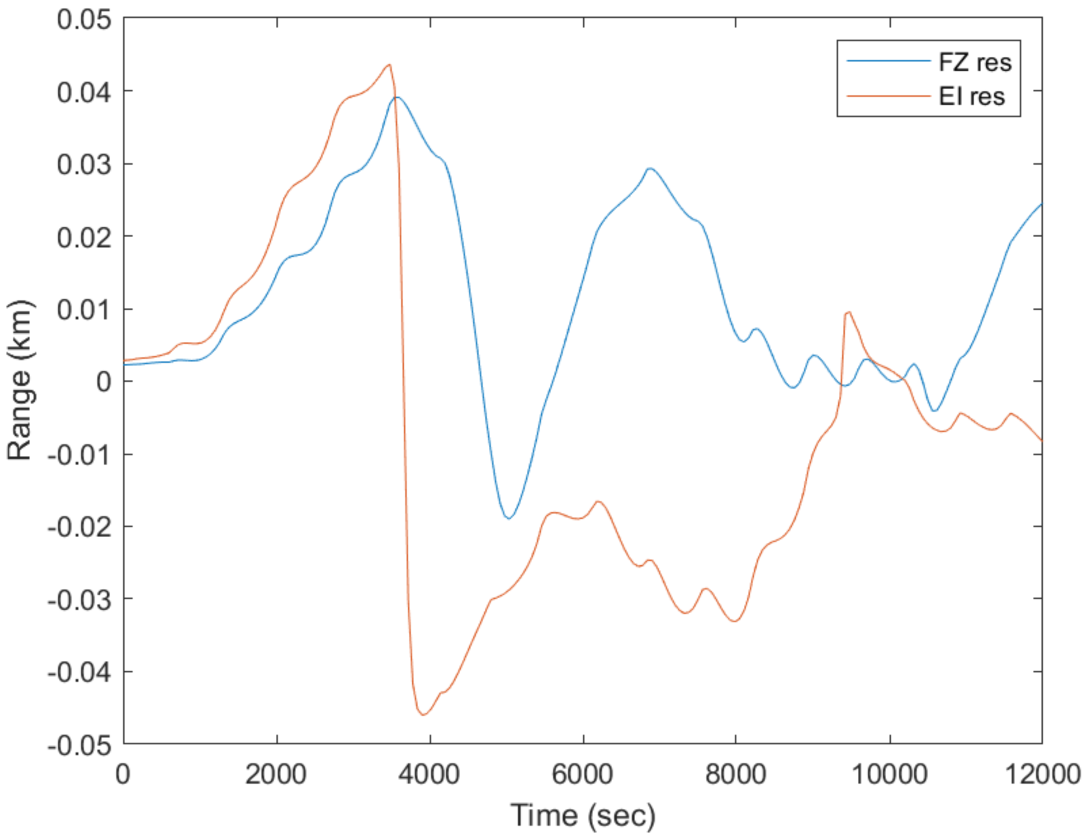
Ideal Observations – Simulated Obs (3/4)

- Range residuals



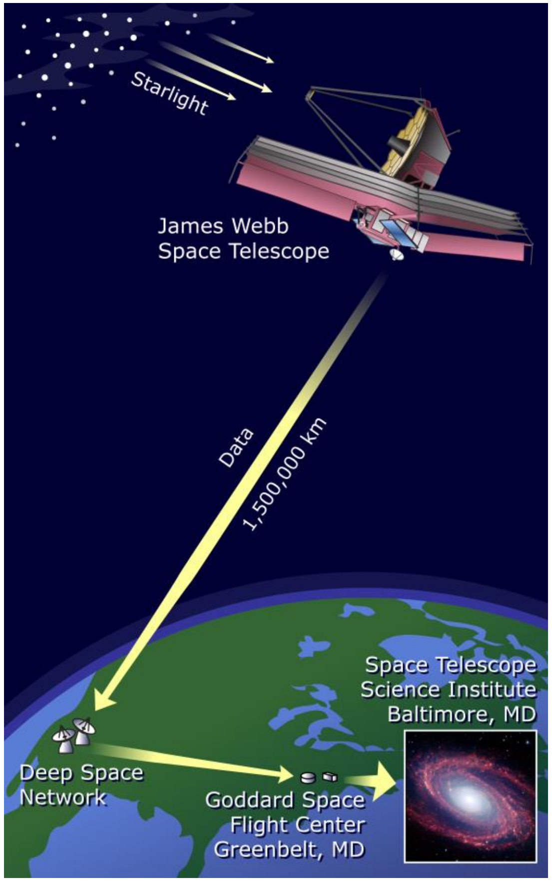
Ideal Observations – Simulated Obs (4/4)

- Correct range residuals



Conceptual Systems – Range (1/4)

- All measurements based on time-of-flight
  - Two-way ranging
    - Passive, reflectance, or active, retransmitted
    - Radar
    - Uplink and downlink path
  - One-way ranging
    - Transmit to receiver
    - Only uplink or downlink
    - GPS



Conceptual Systems – Range (2/4)

- One-way range
  - If clocks are precisely synchronized
$$\tilde{\rho} = c(t_R - t_T)$$
  - Because clock synchronization
    - $\tilde{\rho}$ is related to true range
    - $\tilde{\rho}$ is pseudorange
- GPS use case
  - Time of transmit is predetermined
    - Governed by satellite clock
  - Time of signal arrival measured by independent clock
  - t is clock time
$$t = T + a + b(T - T_0) + \epsilon'$$
    - a is constant offset from true time
    - b is linear clock drift
    - T_0 is some reference time

Conceptual Systems – Range (3/4)

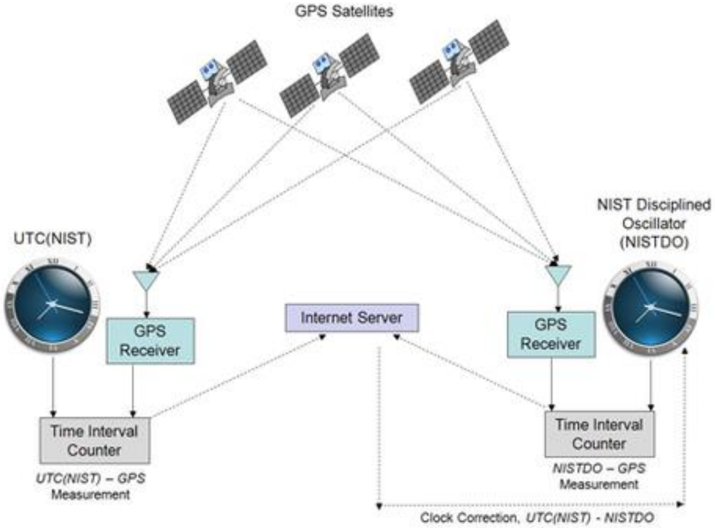
- Applying linear clock model to transmitter and receiver

$$\tilde{\rho} = c(T_R - T_T) + c(a_R - a_T) + c(b_R - b_T)(T - T_0) + \epsilon$$

  - So now we see how psuedorange is related

$$\tilde{\rho} = \rho(T_R, T_T) + c(a_R - a_T) + c(b_R - b_T)(T - T_0) + \epsilon$$

  - An atmospheric term, $\delta\rho_{atm}$ will be necessary to add
- Will need to calculate computed range
  - Obtain residual for orbit determination
  - GPS/NIST precision timing used for NASDAQ
- Use inertial frame to prevent calculation of curved difference



Conceptual Systems – Range (4/4)

- Two-way range
  - Experience same phenomena twice
    - Assume no time delay for retransmission
  - Major advantage
    - Removal of constant offset term, a

$$\rho_{rt} = c(T_R - T_T) + b(T_R - T_T) + \delta\rho_{atm} + \epsilon$$

  - Computed range requires iterative process
    - Satellite signal arrival time is unknown
    1. Find instantaneous ρ at time t_T (assume negligible clock errors)
    2. Signal arrival, $t_a = t_T + \rho/c$
    3. New range, ρ_{new} at t_a, and position of station
    4. Compare ρ_{new} and ρ difference
    - Can estimate by halving instaneous range and accounting for speed of light
    - Same with altimeter

$$h_{avg} = \frac{h_{rt}}{2}$$

$$t_{avg} = t_T + h_{avg}/c$$

Conceptual Systems – Range Rate (1/5)

- Range rate
  - Most range rate systems currently in use are based on single path
  - Two viewpoints
  1. Short duration pulse at known interval
  2. Beacon transmitting signal with known frequency
- Repeated pulse transmission
  - Pulses transmitted at constant interval, δt_T, for sequence, $t_{T1}, t_{T2}, \ldots$
  - Received by ground at $t_{R1}, t_{R2}, \ldots$

$$t_{R1} = t_{T1} + \rho_1/c$$
$$t_{R2} = t_{T2} + \rho_2/c$$
$$t_{R2} - t_{R1} = \delta t_T + (\rho_2 - \rho_1)/c$$

  - Or

$$\delta t = \delta t_T + \delta\rho/c$$

Conceptual Systems – Range Rate (2/5)

- Repeated pulse transmission
  - Three possible cases

1. If $\rho_2 > \rho_1$, satellite moving away, path longer, $\delta t > \delta t_T$
2. If $\rho_2 < \rho_1$, satellite moving closer, path shorter, $\delta t < \delta t_T$
3. If $\rho_2 = \rho_1$, no change in signal path length, $\delta t = \delta t_T$

  - Rewriting to show range-rate measurement

$$\delta t = \delta t_T \left(1 + \left(\frac{\delta \rho}{\delta t_T}\right)/c\right)$$

  - Form of range-rate but only instantaneous as δt_T approaches 0
  - Will need to consider clock errors

Conceptual Systems – Range Rate (3/5)

- Transmitter Beacon (Doppler…)
  - Satellite transmits signal with known frequency, f_T
  - Received signal mixed with reference f_G
    - Receiver designed to count number of cycles between t_{R1} and t_{R2}

$$N_{1,2} = \int_{t_{R1}}^{t_{R2}} (f_G - f_R)\, dt$$

  - Expanding the time terms and assuming f_G is constant

$t_{R1} = t_{T1} + \Delta t_1$, where $\Delta t_1 = \rho_1 / c$ and ρ_1 is range at t_{R1}

$$N_{1,2} = f_G \left[t_{T2} - t_{T1} + \frac{\rho_2 - \rho_1}{c} \right] - \int_{t_{R1}}^{t_{R2}} f_R\, dt$$

$$\int_{t_{R1}}^{t_{R2}} f_R\, dt = \int_{t_{T1}}^{t_{T2}} f_T\, dt = f_T [t_{T2} - t_{T1}]$$

$$N_{1,2} = (f_G - f_T)(t_{T2} - t_{T1}) + f_G (\rho_2 - \rho_1)/c$$

$$\frac{N_{1,2}}{\delta t} = \left(\frac{f_T}{c} \right) \left(\frac{\delta \rho}{\delta t} \right)$$

  - Received frequency depends on range rate

$$f_R = f_T - \frac{N_{1,2}}{\delta t} = f_T \left(1 - \left(\frac{\delta \rho}{\delta t} \right) / c \right)$$

Conceptual Systems – Range Rate (4/5)

- Transmitter Beacon (Doppler…)
  1. If $f_T - f_R < f_T$, moving away, rel. motion (+), apparent freq. is lower than f_T
  2. If $f_T - f_R > f_T$, moving closer, rel. motion is (-), apparent freq. is higher than f_T
  - Instantaneous range rate cannot be measured
    - $N_{1,2}$, measured quantity, related to rate change, $\delta\rho$, over interval, δt
  - Doppler count, zero crossings of $(f_G - f_T)$

Conceptual Systems – Range Rate (5/5)

- Can also form range measurement from $N_{1,2}$
  - Rearranging,

$$\rho_2 = \rho_1 + \frac{N_{1,2}\, c}{f_G} - \frac{c}{f_G}(f_G - f_T)(t_{T2} - t_{T1})$$

    - In this case, ρ_1 is not known so ρ_2 can be treated as a biased range (see pseudorange)
  - In the case when $f_G = f_T$

$$\rho_2 = \rho_1 + \rho_{ph}$$

  - Where $\rho_{ph} = \lambda N_{1,2}$, $\lambda = c/f_T$

Examples for range and range-rate

MEASUREMENT MODELING

Measurement Modeling - Overview

- Two-way ranging (p. 106)

  Consider satellite in equatorial posigrade circular orbit with an altitude of 600 km above a spherical Earth. Assume satellite is 20° in true anomaly past the zenith direction of a two-way ranging station, which places the satellite at 4.3° elevation w/r to the station. Assume a signal is transmitted from the station at $t = 0$.

h	e	i	f	El
600 km	0	0 deg	20 deg	4.3 deg

- Range rate

  Consider the same satellite with a transmitter beacon. Assume the transmitter is operating in the Ka-band at 24.25 GHz.